

# The galaxy stellar mass–star formation rate relation: Evidence for an evolving stellar initial mass function?

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## ABSTRACT

The evolution of the galaxy stellar mass–star formation rate relationship ( $M_*$ –SFR) provides key constraints on the stellar mass assembly histories of galaxies. For star-forming galaxies,  $M_*$ –SFR is observed to be fairly tight with a slope close to unity from  $z \sim 0 \rightarrow 2$ , and it evolves downwards roughly independently of  $M_*$ . Simulations of galaxy formation reproduce these trends, broadly independent of modeling details, owing to the generic dominance of smooth and steady cold accretion in these systems. In contrast, the observed amplitude of the  $M_*$ –SFR relation evolves markedly differently than in models, indicating either that stellar mass assembly is poorly understood or that observations have been misinterpreted. Stated in terms of a star formation activity parameter  $\alpha_{\text{sf}} \equiv (M_*/\text{SFR})/(t_{\text{Hubble}} - 1\text{Gyr})$ , models predict a constant  $\alpha_{\text{sf}} \sim 1$  out to redshifts  $z \sim 4+$ , while the observed  $M_*$ –SFR relation indicates that  $\alpha_{\text{sf}}$  increases by  $\sim \times 3$  from  $z \sim 2$  until today. The low  $\alpha_{\text{sf}}$  (i.e. rapid star formation) at high- $z$  not only conflicts with models, but is also difficult to reconcile with other observations of high- $z$  galaxies, such as the small scatter in  $M_*$ –SFR, the slow evolution of star forming galaxies at  $z \sim 2 - 4$ , and the modest passive fractions in mass-selected samples. Systematic biases could significantly affect measurements of  $M_*$  and SFR, but detailed considerations suggest that none are obvious candidates to reconcile the discrepancy. A speculative solution is considered in which the stellar initial mass function (IMF) evolves towards more high-mass star formation at earlier epochs. Following Larson (1998), a model is investigated in which the characteristic mass  $\hat{M}$  where the IMF turns over increases with redshift. Population synthesis models are used to show that the observed and predicted  $M_*$ –SFR evolution may be brought into general agreement if  $\hat{M} = 0.5(1+z)^2 M_\odot$  out to  $z \sim 2$ . Such IMF evolution matches recent observations of cosmic stellar mass growth, and the resulting  $z = 0$  cumulative IMF is similar to the “paunchy” IMF favored by Fardal et al. (2007) to reconcile the observed cosmic star formation history with present-day fossil light measures.

**Key words:** galaxies: evolution, galaxies: formation, galaxies: high-redshift, cosmology: theory, stars: mass function

## 1 INTRODUCTION

How galaxies build up their stellar mass is a central question in galaxy formation. Within the broadly successful cold dark matter scenario, gas is believed to accrete gravitationally into growing dark matter halos, and then radiative processes enable the gas to decouple from non-baryonic matter and eventually settle into a star-forming disk (e.g. Mo, Mao, & White 1998). However, many complications arise when testing this scenario against observations, as the stellar assembly of galaxies involves a host of other processes including star formation, kinetic and thermal feedback from various sources, and merger-induced activity, all of which are poorly understood in comparison to halo assembly.

A key insight into gas accretion processes is the recent recognition of the importance of “cold mode” accretion (Katz et al. 2003; Birnboim & Dekel 2003; Keres et al. 2005), where gas in-

falling from the intergalactic medium (IGM) does not pass through an accretion shock on its way to forming stars. Simulations and analytic models show that cold mode dominates global accretion at  $z \gtrsim 2$  (Keres et al. 2005), and dominates accretion in all halos with masses  $\lesssim 10^{12} M_\odot$  (Birnboim & Dekel 2003; Keres et al. 2005).

The central features of cold mode accretion are that it is (1) *rapid* (2) *smooth*, and (3) *steady*. It is rapid because it is limited by the free-fall time and not the cooling time; it is smooth because most of the accretion occurs in small lumps and not through major mergers (Murali et al. 2001; Keres et al. 2005; Guo & White 2007); and it is steady because it is governed by the gravitational potential of the slowly growing halo. A consequence of cold mode accretion is that the star formation rate is a fairly steady function of time (e.g. Finlator et al. 2006; Finlator, Davé & Oppenheimer 2007), which results in a tight relationship between the stellar

mass  $M_*$  and the star formation rate SFR. Hence simulations of star-forming galaxies generically predict a tight relationship with  $M_* \propto \text{SFR}$  that evolves slowly with redshift. In detail, a slope slightly below unity occurs owing to the growth of hot halos around higher-mass galaxies that retards accretion.

The stellar mass–star formation rate ( $M_*$ –SFR) relation for star forming galaxies has now been observed out to  $z \sim 2$ , thanks to improving multiwavelength surveys. As expected from models, the relationship is seen to be fairly tight from  $z \sim 0$ – $2$ , with a slope just below unity and a scatter of  $\lesssim 0.3$  dex (Noeske et al. 2007a; Elbaz et al. 2007; Daddi et al. 2007). It also evolves downwards in amplitude to lower redshift in lock step fashion, suggesting a quiescent global quenching mechanism such as a lowering of the ambient cosmic density, again as expected in models. The range of data used to quantify this evolution is impressive: Noeske et al. (2007a) used the AEGIS multi-wavelength survey in the Extended Groth Strip to quantify the  $M_*$ –SFR relation from  $z \sim 1 \rightarrow 0$ ; Elbaz et al. (2007) used GOODS at  $z \sim 0.8 - 1.2$  and SDSS spectra at  $z \sim 0$ ; and Daddi et al. (2007) used star-forming BzK-selected galaxies with *Spitzer*, X-ray, and radio follow-up to study the relation at  $z \sim 1.4 - 2.5$ . In each case, a careful accounting was done of both direct UV and re-radiated infrared photons to measure the total galaxy SFR, paying particular attention to contamination from active galactic nuclei (AGN) emission (discussed in §4). Multiwavelength data covering the rest-optical were employed to accurately estimate  $M_*$ . This broad agreement in  $M_*$ –SFR slope, scatter, and qualitative evolution between model predictions and these data lends support to the idea that cold mode accretion dominates in these galaxies.

Yet all is not well for theory. Closer inspection reveals that the *amplitude* of the  $M_*$ –SFR relation evolves with time in a way that is inconsistent with model expectations. This disagreement is fairly generic, as shown in §2, arising in both hydrodynamic simulations and semi-analytic models, and is fairly insensitive to feedback implementation. The sense of the disagreement is that going to higher redshifts, the observed SFRs are higher, and/or stellar masses lower, than predicted in current models.

The purpose of this paper is to understand the implications of the observed  $M_*$ –SFR relation for our theoretical view of how galaxies accumulate stellar mass. In §3 it is argued that  $M_*$ –SFR amplitude evolution implies a typical galaxy star formation history that is difficult to reconcile with not only model expectations, but also other observations of high- $z$  galaxies. Possible systematic effects that may bias the estimation of SFR and  $M_*$  are considered in §4, and it is argued that none of them are obvious candidates to explain the discrepancy. Finally a speculative avenue for reconciliation is considered, namely that the stellar initial mass function (IMF) has a characteristic mass that evolves with redshift (§5). This model is constrained based on the  $M_*$ –SFR relation in §6. The implications for such an evolving IMF are discussed in §7. Results are summarized in §8. A  $\Lambda$ CDM cosmology with  $\Omega = 0.25$  and  $H_0 = 70$  km/s/Mpc is assumed throughout for computing cosmic timescales.

## 2 $M_*$ –SFR: MODELS VS. OBSERVATIONS

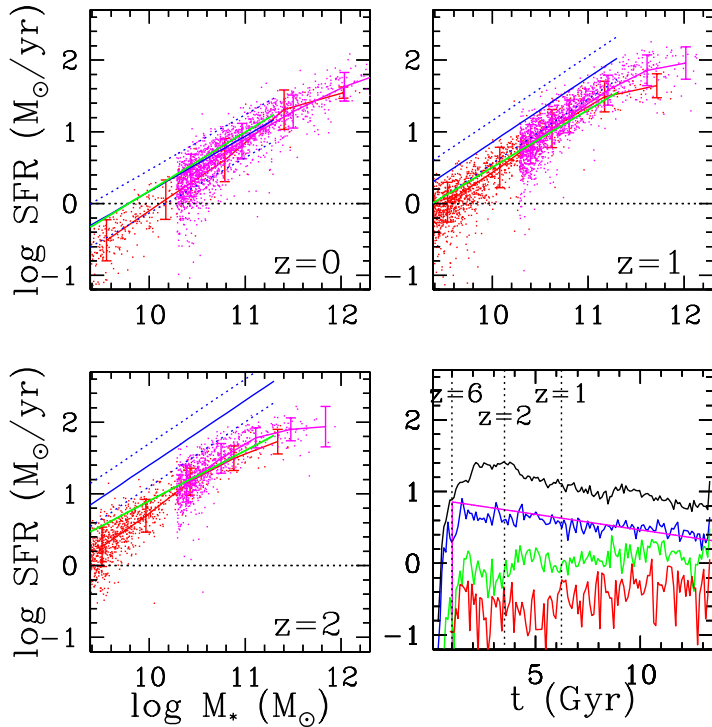
To begin, the observed  $M_*$ –SFR relation is compared with model predictions, in order to understand the origin of the relation. The primary simulations used are cosmological hydrodynamic runs with Gadget-2 (Springel 2005) incorporating momentum-driven outflows, which uniquely match a wide variety

of IGM and galaxy observations (e.g. Oppenheimer & Davé 2006; Davé, Finlator & Oppenheimer 2006; Finlator & Davé 2007). The new simulations include stellar recycling assuming a Chabrier IMF, by returning gas (and metals) into the ambient reservoir, in addition to the usual heating, cooling, and star formation, and feedback recipes (Springel & Hernquist 2003; Oppenheimer & Davé 2006). Outflow parameters are computed based on properties of galaxies identified with an on-the-fly group finder. These and other improvements that go towards making simulations more realistic down to  $z = 0$  are described in Oppenheimer & Davé (2007). Two runs to  $z = 0$  are considered here, each with  $256^3$  dark matter and an equal number of gas particle, in cubic volumes of  $32$  and  $64h^{-1}\text{Mpc}$  (comoving) on a side. The assumed cosmology is  $\Lambda$ CDM with  $\Omega_m = 0.3$ ,  $\Omega_\Lambda = 0.7$ ,  $H_0 = 69$  km/s/Mpc,  $\Omega_b = 0.048$ , and  $\sigma_8 = 0.83$ . The resolved galaxy population, defined here as those with stellar masses greater than  $64$  gas particle masses, is limited by stellar masses of  $2.5 \times 10^9 M_\odot$  and  $2 \times 10^{10} M_\odot$  in the two volumes (this is somewhat more conservative than in Finlator et al. 2006, in order to ensure accurate star formation histories).

To explore the impact of feedback parameters, simulations with two other outflow models are considered, namely one with no outflows, and one with the Springel & Hernquist (2003) “constant wind” outflow model, taken from the suite of outflow runs described in Oppenheimer & Davé (2006). These runs have  $32h^{-1}\text{Mpc}$  box sizes with  $2 \times 256^3$  particles, and they were evolved only down to  $z = 2$ . Their cosmology is slightly different, having  $\sigma_8 = 0.9$  from first-year WMAP parameters, but this is not a large effect at the epochs considered here.

In hydro simulations, star formation rates and stellar masses are obtained by summing over particles in galaxies identified using the group finder SKID (Spline Kernel Interpolative DENMAX; <http://www-hpcc.astro.washington.edu/tools/skid.html>); see Keres et al. (2005) for details. Gas particles that have high enough density to form stars have instantaneous star formation rates computed in Gadget-2; it is this SFR that is summed to obtain the instantaneous SFR of a galaxy. Of course, observed galaxy SFR’s and  $M_*$ ’s are measured quite differently than in simulations, a point that will figure prominently in §4. Note that these hydro simulations do not include any feedback from active galactic nuclei (AGN), or any process that explicitly truncates star formation in massive galaxies, hence they do not reproduce the evolution of passive galaxies. Here the focus is on star-forming galaxies only, with the assumption that galaxies are quickly squelched to become passive and so do not appear in star-forming selected samples (e.g. Salim et al. 2007). Since all galaxies in our simulations would be observationally classified ‘as star-forming, no cuts are made to the simulated galaxy population.

To complement the hydro simulations, results for  $M_*$ –SFR are taken from the Millenium simulation plus semi-analytic model (“Millenium SAM”) lightcones of Kitzbichler & White (2007), as fit by Daddi et al. (2007) and Elbaz et al. (2007). Besides being a completely different modeling approach, the Millenium SAM also includes AGN feedback that broadly reproduces the observed passive galaxy population and its evolution. Additionally, a different semi-analytic model based on the Millenium simulation by De Lucia & Blaizot (2007) was examined, to see if variations in semi-analytic prescriptions can result in significant differences in  $M_*$ –SFR. It was found that the De Lucia & Blaizot (2007) model produced a somewhat shallower slope than the Kitzbichler & White (2007) model, which e.g. at  $z \sim 2$  resulted in a smaller offset from observations at  $M_* \sim 10^{10} M_\odot$ , but a



**Figure 1.**  $M_*$ –SFR relation for simulations versus observations at  $z = 0, 1, 2$ . Red points show a  $32h^{-1}\text{Mpc}$  hydro simulation box, magenta show  $64h^{-1}\text{Mpc}$  box. A running mean (with standard deviations shown as vertical bars) is shown separately for each simulation; agreement in overlap region is excellent. Green lines show results from the Millenium SAM. Blue lines show fits to observations at  $z = 2$  (Daddi et al. 2007), and  $z = 0, 1$  (Elbaz et al. 2007), with dashed lines indicating  $\pm 0.3$  dex scatter (Noeske et al. 2007a). The models’ slopes and scatter are in good agreement, but the amplitude deviates from observations at high- $z$ . **Lower right:** Star formation histories in 100 Myr bins for four star forming galaxies selected from the  $32h^{-1}\text{Mpc}$  run closest to a final  $\log M_* = 11, 10.5, 10, 9.5$  (top to bottom curves). Note the lack of large bursts and the similarity of the star formation history shapes, which yields the prediction of a tight  $M_* \propto \text{SFR}$  (roughly) relation. The magenta line shows the form of the SFH assumed in PEGASE modeling discussed in §6.

larger one at  $M_* \sim 10^{11} M_\odot$ . Since the results are qualitatively similar, only the Kitzbichler & White (2007) models are discussed further. While there are many more semi-analytic models in the literature, and perhaps even some that are in better agreement with the  $M_*$ –SFR relation, it is beyond the scope of this work to consider them all.

Figure 1 (first three panels) shows the  $M_*$ –SFR relation at  $z = 0, 1, 2$  in simulations, compared to parameterizations from observations (blue lines) by Elbaz et al. (2007) ( $z \approx 0, 1$ ) and Daddi et al. (2007) ( $z \approx 2$ ). The observed scatter of 0.3 dex (Noeske et al. 2007a; Elbaz et al. 2007; Daddi et al. 2007) is indicated by the dashed blue lines. It is worth noting that other  $z \sim 2$  observations suggest a larger scatter (Shapley et al. 2005; Papovich et al. 2006), so the observational picture on this is not entirely settled. Finally, the green lines show the Kitzbichler & White (2007) Millenium SAM results, which generally track the hydro simulation results, particularly in amplitude which will be the key aspect.

At  $z = 0$ , the relations are generally in agreement, though the observed slope of  $M_* \propto \text{SFR}^{0.77}$  is somewhat shallower than that predicted by the hydro simulations ( $\approx 0.9$ ). The Millenium

SAM produces excellent agreement at  $z = 0$ , having broadly been tuned to do so. By  $z = 1$ , the predicted hydro simulation slopes are similar to that observed by Elbaz et al. (2007) though steeper than seen by Noeske et al. (2007a), but more importantly the amplitude is low by  $\sim \times 2 - 3$ . At  $z = 2$ , this trend continues, with a  $\sim \times 4 - 5$  amplitude offset. The Millenium SAM has a slope that remains near the  $z = 0$  value, hence it is somewhat shallower than observed at  $z \sim 2$ , but the amplitude evolves slowly much like in the hydro simulations. The rapid evolution in observed  $M_*$ –SFR amplitude has been noted by several authors (Papovich et al. 2006; Noeske et al. 2007a), along with the fact that it is more rapid than in the Millenium SAM (Elbaz et al. 2007; Daddi et al. 2007). The new aspect demonstrated here is that the disagreement persists for current hydrodynamic simulations, and as will be shown in the next section this is true regardless of the main free parameter in such models, namely feedback implementation.

The slope and scatter are a direct reflection of the similarity in the shape of galaxy star formation histories (SFH) among various models and at various masses. For a galaxy observed at time  $t_0$ , neglecting mass loss from stellar evolution,

$$M_*(t_0) = \int_0^{t_0} \text{SFR}(t) dt = \text{SFR}(t_0) \int_0^{t_0} \frac{\text{SFR}(t)}{\text{SFR}(t_0)} dt. \quad (1)$$

The integral depends purely on the *form* of the SFH up to  $t_0$ . So long as the SFH form does not vary significantly with  $M_*$ , the slope of  $M_*$ –SFR will be near unity, and so long as the current SFR is similar to the recent past-averaged one, the scatter will be small. As pointed out by Noeske et al. (2007b), the observed tight scatter of  $\lesssim 0.3$  dex implies that the bulk of star-forming galaxies cannot be undergoing large bursts; this will be discussed further in §3. This is consistent with a fairly steady mode of galaxy assembly as generically predicted by cold mode accretion. Note however that the reverse conclusion does not necessarily hold, in the sense that a slope near unity is not uniquely indicative of a steady mode of galaxy formation; other scenarios (such as the staged galaxy formation model of Noeske et al. 2007b) can be constructed to reproduce a slope near unity, as discussed in §3.

Example galaxy SFHs from the  $32h^{-1}\text{Mpc}$  momentum-driven outflow hydro simulation are shown in the lower right panel of Figure 1, for a range of  $z = 0$  stellar masses from  $10^{11} M_\odot - 10^{9.5} M_\odot$ , in 100 Myr bins. They are remarkably self-similar, and generally quite smooth. The increased variance in smaller galaxies owes to discreteness effects in spawning star particles (these SFHs are computed from the history of galaxy star particles rather than the instantaneous SFR at each time); the true scatter in instantaneous SFR increases only mildly at small masses, from  $\sim 0.2$  dex at  $M_* \gtrsim 10^{11} M_\odot$  to  $\sim 0.3$  dex at  $M_* \gtrsim 10^{9.5} M_\odot$ , as shown in Figure 1.

The early simulated SFHs are characterized by a rapid rise in star formation to  $z \sim 6$ , corresponding to rapid early halo growth (Li et al. 2007). Next, there is a period of roughly constant star formation from  $z \sim 6 \rightarrow 2$ ; this will be the key epoch for the present work. Finally, at  $z \lesssim 2$  the star formation rates decline modestly, with more massive systems showing greater decline. This pattern reflects the changing a balance between the growth of potential wells being able to attract more matter, and cosmic expansion which lowers the ambient IGM density and infall rates. It is qualitatively similar to the SFH inferred for the Milky Way. In the models, the fact that the SFH depends weakly on  $M_*$  gives rise to  $M_* \propto \text{SFR}$ , and the fact that merger-driven starbursts are not a significant growth path for galaxy stellar mass (Keres et al. 2005; Guo & White 2007) gives rise to the

small scatter. In detail a slope slightly below unity occurs because more massive galaxies tend to have their accretion curtailed by hot halos (e.g. Keres et al. 2005), resulting in “natural downsizing” (Neistein, van den Bosch & Dekel 2006).

Note that the SFRs of individual galaxies in simulations do not drop by a large amount over a Hubble time. Even for the most massive star forming galaxies today, the SFR in simulations drops only by a factor of few since  $z = 2$ , and smaller galaxies show essentially no drop. Meanwhile, the amplitude of  $M_*$ -SFR drops by nearly an order of magnitude in the models, and even more in the data. So the amplitude evolution owes predominantly to the growth of  $M_*$  of individual galaxies. In other words, models indicate that the  $M_*$ -SFR relation doesn’t evolve downwards in SFR so much as upwards in  $M_*$ .

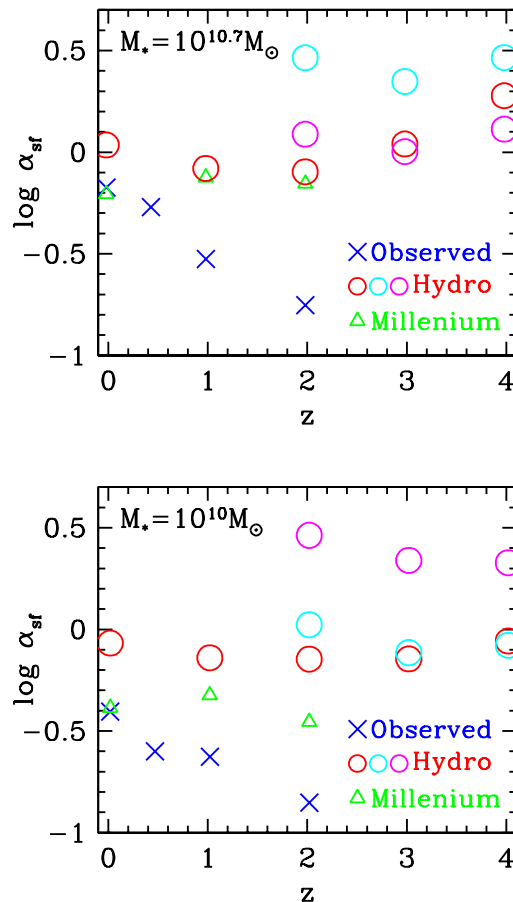
Overall, while the slope and scatter of the  $M_*$ -SFR relation are in broad agreement, there is a stark and generic discrepancy in the amplitude evolution between the models and data. The agreement in slope and scatter is an encouraging sign that our understanding of gas accretion processes is reasonably sound. Examining the discrepancy in amplitude evolution is the focus of the remainder of this paper.

### 3 EVOLUTION OF THE STAR FORMATION ACTIVITY PARAMETER

The amplitude evolution of  $M_*$ -SFR can be recast in terms of the timescale for star formation activity. Models generally predict that galaxies have  $M_*/\text{SFR} \sim t_H$ , where  $t_H$  is the Hubble time, perhaps reduced by 1 Gyr because few stars were forming at  $z \gtrsim 6$ , as seen e.g. in the lower right panel of Figure 1.

A convenient parameterization is provided by the *star formation activity parameter*<sup>1</sup> defined here as the dimensionless quantity  $\alpha_{\text{sf}} \equiv (M_*/\text{SFR})/(t_H - 1\text{Gyr})$ . Physically, this may be regarded as the fraction of the Hubble time (minus a Gyr) that a galaxy needs to have formed stars at its current rate in order to produce its current stellar mass. Note that  $\alpha_{\text{sf}}$  is in general a function of  $M_*$ .

Figure 2 shows  $\alpha_{\text{sf}}(z)$  for simulations and observations, top panel showing values at  $M_* = 10^{10.7} M_\odot$ , bottom showing  $M_* = 10^{10} M_\odot$ . The values for models have been obtained by fitting power laws to simulated galaxies’  $M_*$ -SFR. The momentum-driven outflow hydro simulation results are shown as the red circles, while cyan and magenta circles show results for no winds and the wind model of Springel & Hernquist (2003) at  $z = 2 - 4$ , respectively. The outflow model has only a mild effect on  $\alpha_{\text{sf}}$ , because the *form* of the SFHs are broadly similar regardless of outflows, even though the total amount of stars formed is significantly different (Oppenheimer & Davé 2006). The most dramatic outlier among models is the high-mass bin of the no-wind simulation, where the rapid gas consumption in large galaxies results in lower current star formation rates relative to the past-averaged one (i.e. natural downsizing); this actually makes  $\alpha_{\text{sf}}$  larger and the discrepancy with observations worse. Millenium SAM results are quite similar to the hydro runs, again showing little evolution in  $\alpha_{\text{sf}}$ , although the shallower slope results in a somewhat smaller  $\alpha_{\text{sf}}$  for the lower mass bin. Overall, it is remarkable that regardless of methodology or feedback processes, all models predict  $\alpha_{\text{sf}}$  to be around unity with little evolution from  $z = 4 \rightarrow 0$ , regardless of



**Figure 2.** *Top panel:* Star formation activity parameter  $\alpha_{\text{sf}} \equiv (M_*/\text{SFR})/(t_H - 1\text{Gyr})$  at  $z = 0 - 4$ , evaluated at  $M_* = 10^{10.7} M_\odot$ . Red circles show results from hydro simulations with a momentum-driven outflow model, cyan circles ( $z \geq 2$ ) show hydro simulations with no outflows, and magenta circles ( $z \geq 2$ ) show “constant wind” outflow models (see Oppenheimer & Davé 2006, for details of outflow models). Green triangles show the Millenium SAM results, and blue crosses show the observations by Elbaz et al. (2007,  $z \approx 0, 1$ ), Noeske et al. (2007a,  $z \approx 0.45$ ), and Daddi et al. (2007,  $z \approx 2$ ). In all cases, regardless of modeling technique or outflow implementation,  $\alpha_{\text{sf}}$  is predicted to be around unity or larger at all epochs and at all masses. The observed evolution of  $\alpha_{\text{sf}}$  out to  $z \sim 2$  is markedly discrepant from model predictions. *Bottom panel:* Same as above for  $M_* = 10^{10} M_\odot$ , showing that the trends are broadly insensitive to stellar mass.

stellar mass. This suggests that  $\alpha_{\text{sf}} \sim 1$  is a generic consequence of smooth and steady cold accretion dominating the growth of these galaxies.

In contrast, the observed  $M_*$ -SFR relation yields a rapidly rising  $\alpha_{\text{sf}} = 0.19 \rightarrow 0.31 \rightarrow 0.75$  from  $z = 2 \rightarrow 1 \rightarrow 0$  at  $10^{10.7} M_\odot$ . Values at  $10^{10} M_\odot$  are slightly lower but show similar rapid evolution. The dramatic difference between observed and predicted amplitude evolution either indicates a significant misunderstanding in our basic picture of galaxy assembly, or else that observations have been misinterpreted in some way. Given the currently uncertain state of galaxy formation theory, one might be inclined towards the former, and call into question the entire picture of cold mode-dominated accretion. However, it turns out that the rapidly declining  $\alpha_{\text{sf}}$  also results in uncomfortable conflicts with other *observations* of high-redshift galaxies. To see this, let us discard the idea of cold mode accretion, or any preconceived theoretic

<sup>1</sup> This is similar but not identical to the “maturity parameter” of Scoville et al. (2007).

cal notion, and ask: What possible star formation histories for these galaxies could yield  $\alpha_{\text{sf}}$  rising with time as observed?

One scenario is to postulate that observed high- $z$  galaxies are progressively more dominated by bursts of star formation. Indeed, some models have suggested that bursts may comprise a significant portion of the high- $z$  galaxy population (e.g. Kolatt et al. 1999). However, as Noeske et al. (2007a) points out, this seems unlikely given the tightness of the  $M_*$ -SFR relation. They argue that the observed scatter of 0.3 dex at  $z \sim 1$  implies that most star-forming galaxies have current star formation rates  $\lesssim \times 2 - 3$  of their quiescent value, and that the mean of the relation represents the quiescent “main sequence” of galaxy assembly. Few galaxies between this main sequence and the passive population are seen, arguing for rapid movement between the two populations. At  $z \sim 2$ , the constraints are even more severe, as Daddi et al. (2007) measure a  $1\sigma$  scatter of 0.29 dex (0.16 dex interquartile), while the observed  $\alpha_{\text{sf}} \approx 0.2$  implies they are bursting at  $\sim \times 5$  their quiescent rate. Since these observations are able to probe SFRs down to  $\sim 10 M_\odot/\text{yr}$ , moderate stellar mass ( $M_* \gtrsim 10^{10.5} M_\odot$ ) galaxies with  $\alpha_{\text{sf}} \sim 1$  would have been seen if present, but are not. The only way the burst scenario would work is if all observed galaxies somehow conspire to be bursting at approximately the same level, which seems highly contrived. Hence bursts do not seem to be a viable way to produce low  $\alpha_{\text{sf}}$ .

Another possible scenario is that models simply need to begin forming stars at a later epoch. This would effectively lower  $\alpha_{\text{sf}}$  as defined here, while still maintaining a fairly constant SFR as required by the observed scatter. Theoretically, such a scenario is easier said than done, as the epoch where galaxy formation begins is not a free parameter even in semi-analytic models, but rather is governed by halo formation times that are well-constrained by current cosmological parameters. Such a scenario would require overwhelmingly strong feedback at early epochs, far stronger than even the strong feedback present in e.g. hydro simulations that match observed  $z \sim 4 - 7$  galaxy star formation rates (Davé, Finlator & Oppenheimer 2006; Bouwens et al. 2007), in order for copious dense gas to be present in early halos yet not form stars. Still, in the spirit of discarding any theoretical preconceptions, “delayed galaxy formation” appears to be a viable solution.

However, quantitative consideration of delayed galaxy formation leads to significant problems when confronted by other observations. At  $z = 2$ ,  $\alpha_{\text{sf}} \approx 0.2$  implies that (constant) star formation began roughly 0.5 Gyr prior – which corresponds to  $z \approx 2.3$ ! Of course it cannot be true that *all* galaxies began forming stars at  $z \sim 2.3$ , as many star-forming galaxies are seen at significantly higher redshifts (e.g. Bouwens et al. 2007). Indeed, both the cosmic star formation rate (e.g. Hopkins & Beacom 2006; Fardal et al. 2007), and the rest-UV luminosity function (Reddy et al. 2007) are observed to be fairly constant between  $z \sim 4 \rightarrow 2$ . So it appears galaxies were forming stars quite vigorously at  $z \gg 2.3$ . Even if galaxies smoothly ramped up their star formation to values observed at  $z \sim 2$ , they would still need to start at  $z \approx 2.7$ .

If most galaxies cannot have such a low  $\alpha_{\text{sf}}$ , perhaps the observed systems represent only a small subset of galaxies at  $z \sim 2$ , and the rest are simply unseen. Since current surveys identify both UV-luminous and red, dusty star forming galaxies, it must be that the unseen galaxies are passive, or else so extremely extinguished that they only show up in e.g. sub-millimeter surveys. For the latter case, Reddy et al. (2005) found that the cosmic SFR has a relatively minor contribution from such severely extinguished systems that escape BzK/LBG selection at  $z \sim 2$ , so they are not likely to be the

dominant growth mode for galaxies. Moreover, this actually exacerbates the  $M_*$ -SFR problem, because it provides a mode to form many more stars beyond what is formed quiescently, which would make the expected stellar masses even higher than just from the quiescent mode.

Hence the unseen galaxies must be passive, or forming stars at some undetectably low rate. In the currently popular downsizing scenario (Cowie et al. 1996), these systems would have formed their stars earlier in a rapid mode, and have now attenuated their star formation through some feedback process so as not to be selected in star-forming samples. Perhaps by  $z \sim 2$ , downsizing is already well underway, and the observed galaxies only represent the small fraction of systems that have just recently begun forming stars and will soon be quenched.

While qualitatively sensible, the “unseen passive galaxies” model is once again quantitatively inconsistent with other observations of high- $z$  galaxies. Perez-Gonzalez et al. (2007) used a *Spitzer*/IRAC-selected sample to determine that the star-forming BzK (sBzK) selection technique, i.e. the one employed by Daddi et al. (2007) to measure  $M_*$ -SFR at  $z \sim 2$ , encompasses 88% of their galaxies (2691/3044) from  $1.6 < z < 2.5$ . This is remarkably efficient, significantly more so than e.g. the Lyman break or Distant Red Galaxy selection techniques in obtaining a stellar mass-limited sample. Their stellar mass completeness is estimated to be  $\sim 75\%$  above  $M_* \gtrsim 10^{10.3} M_\odot$  for passively evolving galaxies. Putting these together, this means that conservatively at least two-thirds of all galaxies above this mass threshold are selected as sBzK galaxies, and hence less than one-third are unseen passive galaxies. In contrast, the observed  $M_*$ -SFR relation at  $z \sim 2$  implies  $\alpha_{\text{sf}} \approx 0.2$ , meaning that  $\sim 80\%$  of galaxies should be unseen (passive), or else the duty cycle of such objects should be  $\sim 20\%$ . This assumes galaxies began significant activity  $\sim 1$  Gyr after the Big Bang, and that  $\alpha_{\text{sf}} = 0.2$  at  $z > 2$ ; the latter is again conservative, as an extrapolation would indicate that  $\alpha_{\text{sf}}$  should be even lower at  $z > 2$ . Hence it does not appear that sBzK selection misses nearly enough systems to make the unseen passive galaxies scenario viable to explain  $\alpha_{\text{sf}}(z \approx 2)$ .

Another way to accommodate the low scatter, unity slope, and rapid evolution in  $M_*$ -SFR is to propose that galaxies evolve *along* the  $M_*$ -SFR relation, as proposed by Daddi et al. (2007). At  $z \sim 2$ ,  $\alpha_{\text{sf}} = 0.2$  implies that galaxies must be growing very quickly. Rapid growth means that for any given galaxy, the amplitude of  $M_*$ -SFR is essentially unchanging. Now since  $\text{SFR} \propto M_* = \int \text{SFR} dt$ , an unchanging amplitude implies a solution of

$$\text{SFR} = C \exp(t/\tau) \implies M_* = C\tau \exp(t/\tau) = \tau \text{SFR}. \quad (2)$$

In other words, in this scenario, galaxies have exponentially growing star formation rates and stellar masses! This growth would presumably continue until star formation is truncated by some feedback mechanism, probably when  $M_* \gtrsim 10^{11} M_\odot$ . While at first this “exponentially growing galaxies” idea may seem incredible, of all the scenarios it actually comes closest to reconciling all the high- $z$  galaxy data.

One way to test this scenario is to consider the evolution of the number densities  $n$  of galaxies. The  $e$ -folding time of stellar mass growth is given by  $\tau = M_*/\text{SFR}$ . Assuming (conservatively) that  $\alpha_{\text{sf}} = 0.2$  remains constant at  $z \gtrsim 2$ , then  $\tau \propto t_H - 1 \text{ Gyr}^2$ .

<sup>2</sup> Technically, the fact that  $\tau$  changes with time means that the assumption of a constant  $M_*$ -SFR amplitude in equation 2 is invalid; however, a simple evolutionary model accounting for this yields virtually identical results.

A simple calculation shows that there are e.g. five  $e$ -folding times (or 2.2 decades of growth) from  $z = 3.5 \rightarrow 2$ . For a given stellar mass  $M_*$ ,  $n(M_*)$  at  $z = 2$  should then be similar to  $n(M_*/10^{2.2})$  at  $z = 3.5$  (neglecting mergers). Fontana et al. (2006) measured the number density of galaxies with  $M_* = 10^{11} M_\odot$  at  $z \sim 2$  to be  $7 \times 10^{-4} \text{Mpc}^{-3}$ , and those with  $M_* = 10^{8.8} M_\odot$  at  $z = 3.5$  have  $n = 3 \times 10^{-3} \text{Mpc}^{-3}$  (based on an extrapolation of their STY fits), and stellar mass functions from Elsner et al. (2007) yield similar values. Hence at face value current data suggests that the stellar mass function does not evolve quite as rapidly as expected in this scenario. Still there are substantial uncertainties in the observed stellar mass function at such low masses and high redshifts, so a factor of a few discrepancy is not compelling enough to rule out the exponentially growing galaxies scenario.

Another scenario that has been shown to reproduce  $M_*$ -SFR evolution is the staged galaxy formation model of Noeske et al. (2007b). Here, a delay is introduced in the onset of galaxy formation that is inversely related to the galaxy mass. Hence large-mass objects form early and quickly, while small ones start forming later over longer timescales. This is similar to the exponentially-growing galaxies scenario in the sense that the latter implicitly assumes a sort of delay, because galaxies must accumulate lots of gas in their halo without forming stars in order to have a sufficient gas reservoir to then consume exponentially. Hence it may be possible to constrain staged galaxy formation with similar measurements of the stellar mass function to higher- $z$ ; this is being investigated by K. Noeske (priv. comm.). If either exponentially-growing or staged galaxy formation is correct, this would indicate that star formation at early epochs does not scale with gas density as seen locally (Kennicutt 1998), or else that cold gas is somehow prevented from accumulating in halos as expected from growth of structure.

In summary, the small  $\alpha_{\text{sf}}$  at  $z \sim 2$  implied by the  $M_*$ -SFR amplitude is difficult to reconcile with the observed scatter in  $M_*$ -SFR, the observed evolution of star-forming galaxies to higher redshifts, and the observed fraction of passive galaxies. Taken together, the latter data all point to the idea that observed star-forming galaxies represent the majority of galaxies at intermediate masses ( $\sim 10^{10} - 10^{11} M_\odot$ ) at  $z \sim 2$ , and have been quiescently forming stars over much of a Hubble time, i.e.  $\alpha_{\text{sf}} \sim 1$ . While designer models of stellar assembly cannot be excluded, they would be highly unexpected, and would require substantial revision to our understanding of how gas accumulates and forms stars in hierarchical structure formation. The general difficulty in reconciling the data together with the models' preference for  $\alpha_{\text{sf}} \sim 1$  suggests an uncomfortable situation that bears investigation into alternative solutions.

#### 4 SYSTEMATIC EFFECTS IN $M_*$ AND SFR DETERMINATIONS

Determining  $M_*$  and SFR from broad-band observations is fraught with a notoriously large number of systematic uncertainties that could alter the values of  $\alpha_{\text{sf}}$  inferred from observations. Here it is considered whether such systematics could be responsible for the low value of  $\alpha_{\text{sf}}$  at high- $z$ , particularly focusing on the Daddi et al. (2007) analysis at  $z \sim 2$  since that is where the discrepancies from model expectations and the range of possible systematics are greatest. Star formation rates there are generally estimated by summing rest-UV flux (uncorrected for extinction) with  $24\mu$  flux. For objects that are anomalously high or low in  $24\mu$  to dust-corrected UV SFR estimations, they use the dust-

corrected UV or  $24\mu$  estimate, respectively; however, only one-third of their sample are anomalous in that sense. Meanwhile, stellar masses are estimated from SED fitting: Using multi-band photometry or spectra, one fits model stellar population templates (in their case, Bruzual & Charlot 2003), varying parameters until the best fit is obtained. In detail, Daddi et al. (2007) used a calibration of BzK color to stellar mass based on fitting K20 survey spectra (Fontana et al. 2004). The key assumptions in SED fitting are population synthesis models, dust extinction models, and the form of the star formation history, all of which introduce systematic uncertainties.

Current uncertainties in obtaining accurate stellar masses from rest near-IR data revolve around the poorly known contribution of thermally-pulsating asymptotic giant branch (TP-AGB) stars to the  $K$ -band light (Maraston et al. 2006). This is found to lower the stellar mass estimates for high- $z$  galaxies by a factor of two or more (Maraston et al. 2006; Bruzual 2007). However, the sense would be to *exacerbate* the problem by lowering  $M_*$  and hence further reducing  $\alpha_{\text{sf}}$ , compared with the Bruzual & Charlot (2003) models utilized in Fontana et al. (2004). Hence while uncertainties in population synthesis models are not negligible, they seem unlikely to make a  $\gtrsim \times 3$  difference in the proper direction.

It is possible to hide more stellar mass in objects if one postulates a very old underlying stellar population, in other words by modifying the assumed SFH form. Papovich, Dickinson & Ferguson (2001) found that for UV-selected galaxies, a mass increase of  $\sim \times 3 - 8$  was possible without violating rest-optical constraints. However,  $K$ -selected samples like that of Daddi et al. (2007) are generally redder with more contribution from old stellar light, so the amount that one can hide is more limited (C. Papovich, priv. comm.). More problematic is the fact that doing so requires assuming that the bulk of the galaxy's stars formed very early in the Universe, and hence the current SFR is much lower than at that early epoch. It turns out that this is not a self-consistent solution for reducing  $\alpha_{\text{sf}}$ . This is because if such a SFH history were true, one would expect that the current  $M_*$  would be significantly *more* than obtained by assuming a constant SFH over a Hubble time. This would imply  $\alpha_{\text{sf}} \gg 1$  for those galaxies. Hence even hiding  $5\times$  as much stellar mass as currently inferred would be insufficient, because it would only make  $\alpha_{\text{sf}} \sim 1$ , whereas such a SFH implies  $\alpha_{\text{sf}} \gg 1$ . In order for there to be a large old stellar population while still having a high current SFR, it must be that such galaxies formed many stars early, then sat around not forming stars until a recent flare-up; however, this scenario suffers the same difficulties as the delayed galaxy formation model. Hence one cannot self-consistently alter the SFHs to hide a large amount of very old stars in these galaxies.

Instead, it may be possible to appeal to some systematic reduction in the inferred star formation rate. Why might the true SFR be lower than estimated? One possibility is that the extinction corrections being used for UV-estimated SFRs are too excessive. Now recall that for the majority of galaxies, Daddi et al. (2007) estimated the SFR from the uncorrected UV plus  $24\mu$  flux, so for those cases the extinction correction is not relevant; it is only relevant for the  $\lesssim 25\%$  that are "mid-IR excess" systems. For those, there may be room to alter the extinction law such that the extinction inferred from the UV spectral slope is overestimated. Daddi et al. (2007), like most studies of actively star forming galaxies, use the Calzetti et al. (2000) law. Using instead a Milky Way or LMC law would indeed yield less extinction for a given UV spectral slope, but the typical reduction is less than a factor of two. So it seems unlikely that extinction alone is causing the low  $\alpha_{\text{sf}}$  even in the mi-



nority mid-IR excess objects, unless the extinction law were dramatically different than anything seen in the local universe.

Another possibility is that there is an additional contribution to the rest-UV flux besides star formation. The obvious candidate are AGN, which are common in  $z \sim 2$  star forming galaxies. While Papovich et al. (2006) found that only  $\sim 25\%$  of  $M_* > 10^{11} M_\odot$  galaxies at  $z \sim 2$  have X-ray detectable AGN, Daddi et al. (2007b) identified obscured AGN through their mid-IR excess, and found that they appear in  $\sim 50\%$  of galaxies at  $M_* > 4 \times 10^{10} M_\odot$ , and dominate the overall AGN population at  $z \sim 2$ . Importantly, they also found (in agreement with previous studies, e.g. Papovich et al. 2006) that smaller mass galaxies have a lower AGN fraction, as expected theoretically (Hopkins et al. 2007). Recall that Daddi et al. (2007) used UV fluxes to estimate SFR in such mid-IR excess objects, and by comparison with X-ray data they estimated that such an inferred SFR is unlikely to be wrong by a significant factor. Moreover, since AGN are increasingly rare in smaller mass systems, it would be an odd coincidence if they mimicked the relation  $M_* \propto \text{SFR}^{0.9}$  so tightly. Like the burst model, this represents a fine-tuning problem, where AGN have to contribute a very specific amount to the inferred SFR at each mass scale for no obvious physical reason. This cannot be ruled out, but seems a priori unlikely.

It is also possible that AGN could be present in galaxies that are not mid-IR excess systems. The strong dichotomy in the X-ray properties suggest there is not a continuum of AGN, and that AGNs are limited to mid-IR excess systems (e.g. Daddi et al. 2007b). However, there exist AGN templates where the contribution to rest-UV and rest- $8\mu$  flux are broadly comparable to that in star-forming galaxies (Dale et al. 2006). These could be mistaken for “normal” star-forming systems, whereas in fact the dominant flux is from AGN. Still, once again, to mimic the tight  $M_*$ –SFR relation would require AGN to be contaminating the SFR estimate at a similar level at all stellar masses. Moreover, if they are dominating the UV flux then they should appear as central point sources, but *Hubble*/ACS imaging of sBzK galaxies generally do not show such bright central sources (M. Dickinson, priv. comm.). Hence while AGN are certainly present, it seems unlikely that they are a major contaminant for UV-derived SFRs.

A perhaps even more troublesome issue is the use of rest- $8\mu$  (i.e.  $24\mu$  at  $z \sim 2$ ) as a star formation rate indicator. This is quite uncertain, because the dominant flux at those wavelengths is from polycyclic aromatic hydrocarbon (PAH) emission features, whose nature is not well understood. As Smith et al. (2007) points out, there is more than a factor of two scatter in star formation rate indicators that assume a fixed (local) PAH template. Moreover, owing to PAH features moving in and out of bands, standard photometric redshift uncertainties of  $\Delta z/(1+z) \sim 0.1$  can result in up to an order of magnitude error in inferred SFRs. On the other hand, the tight scatter in the  $z \sim 2$   $M_*$ –SFR relation would seem to limit the amount of scatter owing to PAH feature variations, or else be an indication that UV light dominates over PAH emission (though the latter is not what is seen locally; Smith et al. 2007). Calzetti et al. (2007) notes that  $8\mu$  emission has a substantial metallicity dependence, but it is in the sense that lower metallicities have lower emission, so their SFR would be underestimated by using solar metallicity calibrations. So while it is conceivable that some unknown reason causes PAH emission per unit star formation to be systematically higher in high- $z$  galaxies compared to local ones, thereby causing the SFR to be overestimated, but there are no obvious indications that this would mimic a tight  $M_*$ –SFR relation.

Finally, the observed SFR and  $M_*$ ’s in Daddi et al. (2007) are inferred assuming a Salpeter IMF. Assuming instead a Chabrier

(2003) or Kroupa (2001) IMF would result in shifting *both*  $M_*$  and SFR down by  $\approx 0.15$  dex (Elbaz et al. 2007). Hence to first order it makes no difference as long as the  $M_*$ –SFR slope is around unity (E. Daddi, priv. comm.). Fundamentally, this is because at high- $z$  both stellar mass and star formation rate indicators are driven by light from stars  $\gtrsim 0.5 M_\odot$ , and favored present-day Galactic IMFs generally agree on the shape above this mass. Hence more substantial IMF variations would be required.

In summary, there are no obvious paths to systematically alter  $M_*$  and SFR determination to make up the required  $\sim \times 3 - 5$  difference in  $\alpha_{\text{sf}}$  at  $z \sim 2$ . It is true that many of these systematics cannot be ruled out entirely, and it may be possible that a combination of such effects could explain part or even all of the difference. It is also possible that locally calibrated relations to estimate  $M_*$  and SFR may be substantially different at high- $z$  for unknown and unanticipated reasons. The reader is left to assess the plausibility of such scenarios. The view taken here is that the difficulty in obtaining a straightforward solution motivates the consideration of more exotic possibilities.

## 5 AN EVOLVING IMF?

One possible way to alter  $M_*$ –SFR evolution is to invoke an IMF that is in some direct or indirect way redshift-dependent. The key point is that most direct measures of the star formation rate actually trace high-mass star formation, while stellar mass is dominated by lower-mass stars. Hence by modifying the ratio of high to low mass stars formed, i.e. the shape of the IMF, it is possible to alter the  $M_*$ –SFR relation. The possibility of a varying IMF has been broached many times in various contexts (e.g. Larson 1998; Ferguson et al. 2002; Lacey et al. 2007; Fardal et al. 2007), though typically as a last resort scenario. It is highly speculative as there is no clear-cut evidence at present that supports a time- or space-varying IMF, but it is worth investigating in light of growing observational and theoretical arguments in its favor.

As Kroupa (2001) points out, a universal IMF is not to be expected theoretically, though no variations have been unequivocally detected in local studies of star-forming regions. The average Galactic IMF appears to be well-represented by several power laws, steepest at  $> 1 M_\odot$ , then significantly shallower at  $< 0.5 M_\odot$ , with a turnover at  $< 0.1 M_\odot$  (see review by Kroupa 2007). Forms that follow this include the currently-favored Kroupa (2001) and Chabrier (2003) IMFs. But deviations *are* seen, at least at face value. The nearby starburst galaxy M82 appears to have an IMF deficient in low-mass stars (Rieke et al. 1980, 1993). A similarly bottom-light IMF is inferred in the highly active Arches cluster near the Galactic center, suggesting that vigorous star formation makes the IMF top-heavy (e.g. Figer 2005). Also, the young LMC cluster R136 shows a flattening in the IMF below  $\sim 2 M_\odot$  (Sirianni et al. 2000). There are hints that presently active star clusters form more low-mass stars compared to the disk-averaged 5 Gyr-old IMF (Kroupa 2007), suggesting that the IMF was weighted towards heavier stars at lower metallicities and/or earlier times. Even if the IMF shape is universal, larger starforming regions may produce more high-mass stars simply because they have enough material to aggregate into larger stars (Weidner & Kroupa 2006).

Hence there are hints of IMF variations, albeit controversial, and interestingly they consistently go in the direction of having a higher ratio of massive to low-mass stars in conditions similar to those in high- $z$  galaxies. Compared to present-day star-forming systems, galaxies at high- $z$  have far higher star formation

surface densities than the Galactic disk (Erb et al. 2006c), have higher gas content and hence presumably more massive star-forming clouds (Erb et al. 2006b), and have somewhat lower metallicities (Savaglio et al. 2005; Erb et al. 2006a). Larson (1998) and Fardal et al. (2007) list other pieces of circumstantial evidence that favor early top-heavy IMFs. So while there are no direct observational evidences or strong theoretical arguments for it, it is not inconceivable that the IMF in high- $z$  galaxies may be more top-heavy or bottom-light<sup>3</sup> than the present-day Galactic IMF.

Such an IMF would have three main effects on the  $M_*$ –SFR relation: It would increase the output of UV flux per unit stellar mass formed; it would cause more stellar mass loss from stellar evolution; and the mass recycling would provide a larger gas reservoir with which to form stars. All three effects go towards reconciling theoretical expectations with observations of the  $M_*$ –SFR relation. The SFR inferred by assuming a standard IMF would be an overestimate, since many fewer low-mass stars are formed per high-mass star. Increased recycling losses would lower the theoretical expectations for the amount of stellar mass remaining in these galaxies, bringing them more into line with observed stellar masses. Recycled gas would increase the available reservoir for new stars, effectively delaying star formation in galaxies and pushing models towards the “delayed galaxy formation” scenario. Hence even modest evolution in the IMF could in principle reconcile the observed and theoretically-expected  $\alpha_{\text{sf}}$  evolution.

## 6 A SIMPLE IMF EVOLUTION MODEL

How is the IMF expected to evolve with time? This is difficult to predict since no *ab initio* theory for the IMF exists today. But a simple conjecture by Larson (1998, 2005) may have some empirical value. He notes that there is a mass scale of  $\sim 0.5M_\odot$  below which the Galactic IMF becomes flatter (Kroupa, Tout, & Gilmore 1993; Kroupa 2001; Chabrier 2003), and at which point the mass contribution per logarithmic bin ( $MdN/d\log M$ ) is maximized. Larson uses simple thermal physics arguments to show that this *characteristic IMF mass scale* ( $\hat{M}_{\text{IMF}}$ ) is related to the minimum temperature  $T_{\text{min}}$  of molecular clouds, measured today to be around 8 K. In that case, if the temperature in the interstellar media (ISM) of high- $z$  galaxies is higher, this characteristic mass will shift upwards, resulting in an IMF that is more bottom-light.

To generate a crude IMF evolution model, let us assume that the  $\hat{M}_{\text{IMF}} \propto (1+z)^\gamma$ , where  $\gamma$  is some unknown parameter whose value is set by the various effects above. Larson (1985) predicted analytically that  $\hat{M}_{\text{IMF}} \propto T_{\text{min}}^{3.35}$ , while recent numerical experiments by Jappsen et al. (2005) found a less steep scaling of  $T_{\text{min}}^{1.7}$ . So for example an evolution that scaled purely with CMB temperature would yield  $\gamma = 1.7$ . However, other factors may cause an increased ISM temperature, e.g., elevated disk SFR rates causing more supernova heat input; lower cooling rates owing to lower metallicities; and/or photoionization from a stronger metagalactic UV flux (e.g. Scott et al. 2002).

The approach adopted here is to treat  $\gamma$  as a free parameter, to be constrained using the observed evolution of  $\alpha_{\text{sf}}$ . The assumption is that  $\alpha_{\text{sf}}(z=0)$  reflects the present-day Galactic IMF, while

$\alpha_{\text{sf}}(z > 0)$  is inferred to be lower purely because of the assumption of a non-evolving IMF, whereas in reality  $\alpha_{\text{sf}}(z)$  (at any given mass) is constant as expected in models.

An “evolving Kroupa” IMF is defined in which

$$\hat{M}_{\text{IMF}} = 0.5(1+z)^\gamma M_\odot. \quad (3)$$

Above  $\hat{M}_{\text{IMF}}$ , the IMF has a form  $dN/d\log M \propto M^{-1.3}$ , while below it scales as  $M^{-0.3}$ . Note that the current Galactic IMFs also have a turn-down at masses below  $0.1M_\odot$ ; the present analysis provides no constraints in this regime, as the stellar mass formed there is a small fraction of the total. The exact form of the IMF is not critical here; Kroupa (2001) is chosen for its convenient explicit parameterization in terms of  $\hat{M}_{\text{IMF}}$ , but the analysis below could equally well have been done with the similar Chabrier IMF.

The goal of the evolving IMF is to increase the ratio of the star formation rate of *high-mass* stars ( $\text{SFR}_{\text{hiM}}$ ) to total stellar mass formed, such that the inferred  $\alpha_{\text{sf}}(z)$  is non-evolving. Note that different probes such as UV luminosity, H $\alpha$  emission, and mid-IR emission trace slightly different regimes of high-mass star formation, but so long as the high-mass IMF slope remains fixed, the relative ratios of such emission should be constant. So for example, in order to produce no evolution in  $\alpha_{\text{sf}}$  from  $z \sim 0 \rightarrow 2$  at  $10^{10}M_\odot$ , the factors by which  $f \equiv \text{SFR}_{\text{hiM}}/M_*$  must be raised are 1.9 at  $z=1$  and 3.3 at  $z=2$ , relative to a standard Kroupa IMF (cf. Figure 2). Hence  $\gamma$  must be chosen to match these factors.

What value of  $\gamma$  would produce such evolution? To determine this, the PEGASE.2 (Fioc & Rocca-Volmerange 1997) population synthesis model is employed, taking advantage of its feature allowing user-definable IMFs. PEGASE outputs a variety of quantities as a function of time from the onset of star formation, given an input star formation history. The assumed SFH is taken to be exponentially-declining with a 10 Gyr decay time and starting 1 Gyr after the Big Bang, in order to approximate a typical model SFH. An example of such a SFH is shown as the magenta line in the lower right panel Figure 1, normalized to a star formation rate of  $2M_\odot\text{yr}^{-1}$  today; as is evident, it is quite similar to typical SFHs from the simulations, at least for moderate stellar masses. A constant SFH was also tried, and the results were not significantly different. Besides the input SFH, all other parameters are taken as the PEGASE default values. No dust extinction is assumed.

The PEGASE output quantities specifically used here are the stellar mass  $M_*$  formed, and the un-extincted 1650Å luminosity ( $L_{1650}$ ). The latter is used as a proxy for  $\text{SFR}_{\text{hiM}}$ . Using these, it is possible to compute  $f$  for any given assumed IMF at any given time. Note that PEGASE accounts for both stellar mass accumulation and mass loss owing to stellar evolution, but does not account for the impact of recycled gas. By varying  $\hat{M}_{\text{IMF}}$ , it is possible to estimate  $f$  for any assumed  $\gamma$ , and thereby determine what value of  $\gamma$  would yield no  $\alpha_{\text{sf}}$  evolution.

In practice, determining  $f \equiv \text{SFR}_{\text{hiM}}/M_*$  at any epoch requires integrating the mass growth up to that epoch for an evolving IMF. To do so, first PEGASE models are computed for a wide range of  $\hat{M}_{\text{IMF}}$  values. Then the following procedure is used:

- (1) A value of  $\gamma$  is selected;
- (2) the following integral is computed:

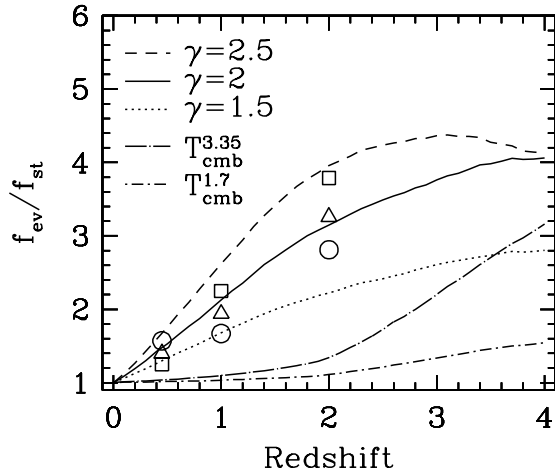
$$M_*(t_0) = \int_0^{t_0} \frac{\Delta M}{\Delta t}(\hat{M}_{\text{IMF}}, t) dt, \quad (4)$$

where  $\Delta M/\Delta t$  is the stellar mass growth rate at time  $t$  taken from PEGASE models interpolated to the appropriate  $\hat{M}_{\text{IMF}}$  as given in equation 3;

- (3)  $L_{1650}$  is taken at time  $t$  from PEGASE, and  $f_{\text{ev}}(t_0) =$

<sup>3</sup> The distinction between top-heavy and bottom-light IMFs is mostly semantic. Here the convention is used that top-heavy means a high-mass slope that is less steep than the local Salpeter value, while bottom-light means that the high-mass slope remains similar but low-mass stars are suppressed.





**Figure 3.** Evolution of the ratio of high-mass star formation rate to stellar mass, for the evolving Kroupa IMF (equation 3) relative to a standard Kroupa IMF, normalized to unity at  $z = 0$ . The curves are computed from PEGASE.2 models for different values of  $\gamma$ , as described in §6. The data points are from fits to the observed  $M_*$ –SFR relation at  $z = 0.45, 1, 2$  (described and shown in Figure 2) at stellar masses of  $10^{10.5} M_{\odot}$  (squares),  $10^{10} M_{\odot}$  (triangles), and  $10^{9.5} M_{\odot}$  (circles). The required evolution is reasonably well fit by  $\gamma = 2$ , i.e.  $\hat{M}_{\text{IMF}} = 0.5(1+z)^2 M_{\odot}$ . Long and short dashed-dot models show results for the Larson (1998) scenario where the minimum temperature of molecular clouds is set by the CMB temperature, and  $\hat{M}_{\text{IMF}} \propto T_{\text{min}}^{\beta}$ , where  $\beta = 3.35$  (Larson 1985) and  $\beta = 1.7$  (Jappsen et al. 2005), respectively. Because the CMB temperature does not exceed the local minimum temperature in molecular clouds of 8 K until  $z \gtrsim 2$ , this scenario does not yield substantial evolution at  $z \lesssim 2$ .

$L_{1650}(t_0)/M_*(t_0)$  is computed (the subscript “ev” stands for “evolving”);

(4) Similarly,  $f_{\text{st}}(t_0)$  is computed for a standard Kroupa IMF, where  $\hat{M}_{\text{IMF}} = 0.5 M_{\odot}$ ;

(5) The ratio  $f_{\text{ev}}(t_0)/f_{\text{st}}(t_0)$  is computed, and compared to the factors needed to produce no  $\alpha_{\text{sf}}$  evolution.

The results of this procedure are shown in Figure 3, namely  $f_{\text{ev}}/f_{\text{st}}$  as a function of redshift for several values of  $\gamma$ . The values by which the  $M_*$ –SFR relation needs to be adjusted to produce no  $\alpha_{\text{sf}}$  evolution at  $z = 0.45, 1, 2$  are indicated by the symbols: Squares are for  $M_* = 10^{9.5} M_{\odot}$ , triangles for  $10^{10} M_{\odot}$ , and circles for  $10^{10.5} M_{\odot}$ . These are computed from published fits to  $M_*$ –SFR (Noeske et al. 2007a; Elbaz et al. 2007; Daddi et al. 2007). Because the  $M_*$ –SFR relation becomes shallower with time, the required adjustment factors are generally smaller at lower masses.

Figure 3 shows that  $\gamma = 2$  provides a reasonable fit to the required IMF evolution for typical star-forming galaxies at  $M_* = 10^{10} M_{\odot}$ . The implied values of  $\hat{M}_{\text{IMF}}$  are  $2 M_{\odot}$  and  $4.5 M_{\odot}$  at  $z = 1, 2$ , respectively. Note that there is no a priori reason why a single value of  $\gamma$  should fit at all redshifts considered, so the good fit supports the form of  $\hat{M}_{\text{IMF}}$  evolution assumed in equation 3.

As a check, an independent method for determining  $\hat{M}_{\text{IMF}}$  evolution was explored that is less comprehensive, but conceptually more straightforward. Here, the high-mass star formation rate is assumed to be unchanged, while observations of stellar mass are assumed to reflect stars near the main sequence turnoff mass ( $M_{\text{turnoff}}$ ), since red optical light is dominated by giant stars. For an evolving Kroupa IMF, to obtain a reduction of a factor of  $x$  in the amount of stars produced at  $M_{\text{turnoff}}$ , it is easy to show that  $\log \hat{M}_{\text{IMF}} = \log M_{\text{turnoff}} + \log x$ .  $M_{\text{turnoff}}$  may be estimated by

noting that stellar lifetimes scale as  $M^{-3}$ , and that the Sun has a lifetime of 10 Gyr; this yields  $M_{\text{turnoff}}^3 = 10/(t_H - 1)$ , ( $t_H$  in Gyr), which is  $1.24 M_{\odot}$  and  $1.58 M_{\odot}$  at  $z = 1, 2$ , respectively. Inserting values of  $x = 1.9, 3.3$  at  $z = 1, 2$  yields  $\hat{M}_{\text{IMF}} = 2.4$  and  $5.2 M_{\odot}$ . These values are quite similar to those obtained from PEGASE modeling, showing that the results are not critically dependent on details of PEGASE.

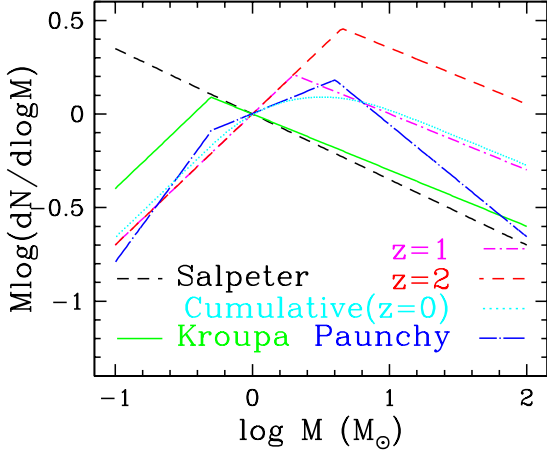
The same procedure can be applied to the scenario proposed by Larson (1998), where the CMB temperature is solely responsible for setting a floor to the ISM temperature. In that case,  $T_{\text{min}} = \text{MAX}[T_{\text{CMB}}, 8\text{K}]$ , and  $\hat{M}_{\text{IMF}} \propto T_{\text{min}}^{\beta}$ . Larson (1985) predicted  $\beta = 3.35$ , while Jappsen et al. (2005) found  $\beta = 1.7$ . The results of this scenario for these two values of  $\beta$  are shown as the long and short dot-dashed lines, respectively, in Figure 3. As is evident, this IMF evolution is not nearly sufficient to reconcile the observed and predicted  $\alpha_{\text{sf}}$  evolution out to  $z \sim 2$ , mainly because  $T_{\text{CMB}}$  only exceeds 8 K at  $z > 1.93$ , and hence  $\hat{M}_{\text{IMF}}$  does not change from  $z \sim 0 - 2$ .

Note that since CMB heating of the ISM is subdominant at all redshifts, the assumed form of IMF evolution (eq. 3, scaling as  $T_{\text{CMB}}^{\gamma}$ ) is not physically well motivated. While the evolution out to  $z \sim 2$  is well-fit by such a form, it could be that the form changes at higher  $z$ , or that it should be parameterized as some other function of  $z$ . For instance, if it is the vigorousness of star formation activity that determines  $\hat{M}_{\text{IMF}}$ , perhaps IMF evolution actually reverses at very high redshifts. For lack of better constraints, the form for IMF evolution in equation 3 will be assumed at all  $z$ , but this should be taken as an illustrative example rather than a well-motivated model.

This analysis also makes predictions for  $\alpha_{\text{sf}}$  at  $z > 2$  that would be inferred assuming a standard (non-evolving) IMF. For instance, Figure 3 implies that at  $10^{10} M_{\odot}$ ,  $\alpha_{\text{sf}}(z = 3) = 0.13$  and  $\alpha_{\text{sf}}(z = 4) = 0.12$ . Notably, the evolution slows significantly at  $z \gtrsim 3$ , and actually reverses at  $z \gtrsim 4$ , though the simple SFH assumed in the PEGASE modeling may be insufficiently realistic to yield valid results at  $z \gtrsim 4$ . Any such extrapolation of this IMF evolution should be made with caution, as the  $M_*$ –SFR relation only constrains it out to  $z \sim 2$ . However, in the next section this will be compared to observations that suggest that such IMF evolution may be reasonable out to  $z \sim 3+$ .

Figure 4 illustrates the evolving IMF, in terms of stellar mass formed per unit logarithmic mass bin. A Salpeter IMF is shown for reference, and evolving Kroupa IMFs are shown at  $z = 0, 1, 2$  with characteristic masses  $\hat{M}_{\text{IMF}} = 0.5 M_{\odot}, 2 M_{\odot}$ , and  $4.5 M_{\odot}$ , respectively. As expected, the evolution results in more high mass stars compared to low mass ones at higher redshifts. While such a dramatic increase in  $\hat{M}_{\text{IMF}}$  may seem surprising, recall that studies of local highly active star-forming regions suggest a characteristic mass scale of a few  $M_{\odot}$  (Rieke et al. 1993; Sirianni et al. 2000; Figer 2005). Note that an IMF with a steeper high-mass slope (e.g. Miller & Scalo 1979; Kroupa, Tout, & Gilmore 1993) would result in even stronger IMF evolution being required than in the Kroupa (2001) case.

In summary, an evolving Kroupa IMF whose characteristic mass increases with redshift as  $\hat{M}_{\text{IMF}} = 0.5(1+z)^2 M_{\odot}$  is able to alter the observed  $\alpha_{\text{sf}}$  evolution from  $z \sim 2 \rightarrow 0$  into one with approximately no evolution, as predicted by models or inferred from complementary observations. The precise IMF shape is not well constrained; it is certainly possible that an IMF with different behavior, for instance an evolving high-mass slope (e.g. Baldry & Glazebrook 2003), could produce the same  $\alpha_{\text{sf}}$  evolution. The only requirements are that it produces the correct ratio of high-mass stars relative to low-mass stars as a function of redshift,



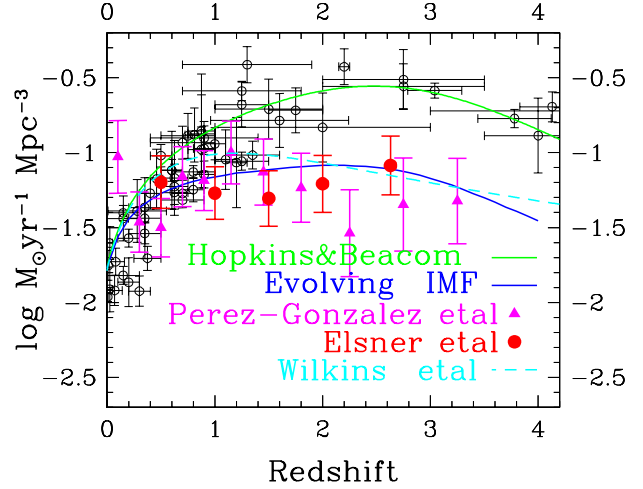
**Figure 4.** Initial mass functions, multiplied by  $M$  to show the mass formed per unit logarithmic mass bin, and arbitrarily normalized at  $1 M_{\odot}$ . Salpeter (dashed) and Kroupa (2001) (green solid) IMFs are shown; Chabrier is similar to Kroupa. Evolving Kroupa IMFs are shown at  $z = 1$  (magenta dot-dashed) and  $z = 2$  (red dashed), with characteristic masses  $\tilde{M}_{\text{IMF}} = 2 M_{\odot}$  and  $4.5 M_{\odot}$ , respectively, as required to remove evolution in  $\alpha_{\text{sf}}$  (see §6). The cumulative  $z = 0$  IMF (cyan dotted) shows the summation of an evolving Kroupa IMF with  $\tilde{M}_{\text{IMF}} = 0.5(1+z)^2$  over the history of the Universe, assuming an evolving IMF cosmic star formation history as shown in Figure 5. Also shown is the “paunchy” IMF (blue) favored by Fardal et al. (2007) for reconciling the cosmic star formation history, the present-day  $K$ -band luminosity density, and extragalactic background light constraints. The cumulative  $z = 0$  evolving IMF is similar to the paunchy IMF, showing that it would go well towards reconciling fossil light data with the integrated cosmic SFH (see §7).

and that (unlike e.g. the merger-induced top-heavy IMF suggested by Lacey et al. 2007) it applies to the bulk of star forming galaxies at any epoch.

## 7 IMPLICATIONS OF AN EVOLVING IMF

What are the observational implications of such an evolving IMF at early epochs? Surprisingly few, as it turns out, so long as the massive end of the IMF remains unaffected. This is because, as mentioned before, virtually all measures of high- $z$  star formation, be they UV, IR or radio, trace light predominantly from high-mass stars. Feedback energetics and metallicities likewise reflect predominantly massive star output. Hence so long as an IMF preserves the same high mass star formation rate, it is expected to broadly preserve the successes of understanding the relationship between the rest-UV properties of galaxies (e.g. Davé, Finlator & Oppenheimer 2006; Bouwens et al. 2007), cosmic metal pollution (e.g. Oppenheimer & Davé 2006; Davé & Oppenheimer 2007), feedback strength, and Type II supernova rates.

The main difference caused by such IMF evolution is in stellar mass accumulation rates. High- $z$  measures of stellar mass are therefore critical for testing this type of scenario. Drory et al. (2005) determined the stellar mass function evolution from a  $K$ -band survey out to  $z \sim 5$ , but at even moderately high redshifts the rest-frame light is actually quite blue, so their stellar mass estimates implicitly involves a significant IMF assumption. The high- $z$  stellar mass–metallicity relation (Erb et al. 2006a) has the potential to test the IMF; if the evolving IMF is correct, the good agreement found



**Figure 5.** Cosmic star formation history. Data points are from Hopkins (2004); best-fit relation to an updated compilation from Hopkins & Beacom (2006) is shown as the green line. A schematic representation of the cosmic star formation history assuming an evolving IMF with  $\gamma = 2$  is shown as the blue line, which is simply the Hopkins & Beacom (2006) fit divided by the ratio of high-mass star formation to stellar mass growth for an evolving IMF versus a constant one, namely the solid ( $\gamma = 2$ ) line in Figure 3. Magenta and red points show the result of differentiating the cosmic stellar mass density measurements as a function redshift by Perez-Gonzalez et al. (2007) and Elsner et al. (2007), respectively. Cyan dashed line shows the fit to cosmic star formation rate inferred from a compilation of cosmic stellar mass densities by Wilkins, Trentham & Hopkins (2007). For ease of comparison, all data here are shown here assuming a Salpeter IMF. The evolving IMF case is in good agreement with direct measures of stellar mass growth.

versus recent models (Finlator & Davé 2007; Brooks et al. 2007; Kobayashi, Springel & White 2007) would perhaps not stand, and in particular many more metals would have to be driven out of galaxies of a given  $M_*$  via outflows (i.e. the “missing metals” problem would get worse; Davé & Oppenheimer 2007). Type Ia supernova rates at high- $z$  would be lowered; the apparent turn-down in Type Ia rates at  $z \gtrsim 1$  argued by Hopkins & Beacom (2006) as being indicative of a long Type Ia delay time of 3 Gyr, may instead be accommodated via a more canonical short delay time coupled with an evolving IMF. These are examples of the types of observations that could potentially constrain IMF evolution.

The star formation rates inferred for high-redshift galaxies by assuming present-day IMFs would have to be revised downwards for an evolving IMF. A quantitative estimate of the reduction in star formation rates from the standard to evolving IMF case is provided by the ratio  $f_{\text{ev}}/f_{\text{st}}$  as a function of redshift from Figure 3. Specifically, the “true” cosmic star formation rate (assuming the evolving IMF is correct) is the one currently inferred assuming a non-evolving IMF, divided by the curve for a particular value of  $\gamma$ , say  $\gamma = 2$ .

Figure 5 shows the cosmic star formation history with data points compiled by Hopkins (2004), and the fit by Hopkins & Beacom (2006). It also schematically illustrates the effect of an evolving IMF by dividing the Hopkins & Beacom (2006) fit by the  $f_{\text{ev}}/f_{\text{st}}$  curve for  $\gamma = 2$ . Note that Hopkins & Beacom (2006) used a Salpeter IMF to infer star formation rates, so these correction factors (derived for a Kroupa IMF) are not exact, but should be reasonably close. Figure 5 shows that the cosmic SFR peak shifts to later epochs in the evolving IMF case. The total stel-

lar mass formed in the Universe is reduced by 55% by  $z = 0$  and 75% by  $z = 2$ , compared to a standard IMF.

Interestingly, some current observations seem to *prefer* an IMF at early epochs that is more top-heavy or bottom-light, and perhaps a later epoch of cosmic star formation than generally believed. Hopkins & Beacom (2006) noted that the integral of the cosmic star formation history exceeds the stellar mass currently detected by a factor of two, assuming a Salpeter IMF. Borch et al. (2006) also noted such tension, but suggested that switching from Salpeter to a Chabrier or Kroupa IMF might go significantly towards reconciling the difference; little evidence exists for IMF evolution to  $z \lesssim 1$  (Bell et al. 2007). But to higher redshifts, observations by Rudnick et al. (2006) of stellar mass density evolution to  $z \sim 3$  suggest that mass buildup is peaked towards significantly later epochs than in current models that broadly match the observed cosmic star formation history. Baldry & Glazebrook (2003), updating an analysis done by Madau, Pozzetti, & Dickinson (1998), found that the cosmic SFH and present-day cosmic luminosity density are best reconciled using an IMF slope that is slightly more top-heavy than Salpeter ( $-1.15$  vs.  $-1.35$ ), and significantly more top-heavy than e.g. Kroupa, Tout, & Gilmore (1993). Heavens et al. (2004) suggests from an archaeological analysis of SDSS spectra that the cosmic star formation rate peaked at  $z \sim 0.6$  rather than the more canonical  $z \sim 2$ . Van Dokkum & van der Marel (2007) find that it is easier to understand early-type fundamental plane evolution at  $z \lesssim 1$  with a slightly top-heavy IMF (as also argued by Renzini 2005). Intracluster medium metallicities indicate that stars formed in clusters at early epochs either had higher yields than today, or else formed with a top-heavy IMF (Portinari et al. 2004). Lucatello et al. (2005) and Tumlinson (2007) suggest that the abundance of carbon-enhanced metal-poor stars is indicative of a more top-heavy IMF at early times in our Galaxy. Chary (2007) finds that an IMF slope more top-heavy than Salpeter is required to produce enough early photons to reionize the Universe. While it should be pointed out that a large number of observations are consistent with a universal IMF, whenever a discrepancy exists, it is invariably in the sense of favoring an IMF with more massive stars at early times.

Fardal et al. (2007) did perhaps the most careful work on trying to reconcile fossil light measures with redshift-integrated measures of stellar mass. Extending the analyses of Baldry & Glazebrook (2003) and Hopkins & Beacom (2006) by including extragalactic background light (EBL) constraints, they found that *no* standard IMF (Kroupa, Chabrier, Salpeter, etc.), is able to reconcile at the  $\sim 3\sigma$  level the cosmic star formation history, the present-day  $K$ -band luminosity density, and the EBL. They examined how the IMF might be altered to make these observations self-consistent, and found that what was required was an IMF that had an excess at intermediate masses ( $\sim$  few  $M_\odot$ ) over standard IMFs. This IMF need not be universal, but instead could be a cumulative IMF of all stars that have formed in the Universe, since it was obtained from fossil light considerations. An example of a concordant IMF is their “paunchy” IMF, shown as the blue dot-dashed line in Figure 4.

Using the evolving Kroupa IMF, it is possible to compute a cumulative IMF of all star formation in the Universe. This is done by integrating the evolving IMF over cosmic time weighted by the cosmic star formation history, which is taken to be the evolving IMF one (blue line) from Figure 5. The result is shown as the dotted cyan line in Figure 4. It is similar to the paunchy IMF of Fardal et al. (2007), showing elevated formation rates of intermediate mass stars, though it is slightly more top-heavy. This

comparison is preliminary because the EBL and current  $K$ -band light trace stellar output differently than the cosmic star formation history. Still, it is likely that this evolving IMF will go significantly towards reconciling the various observations considered in Fardal et al. (2007).

Van Dokkum (2007) recently argued for a more top-heavy IMF at early times from the evolution of colors and mass-to-light ratios of early-type galaxies in clusters from  $z \sim 0.8 \rightarrow 0$ . Parameterizing the evolution in terms of the characteristic mass of a Chabrier IMF, he favored a characteristic mass that evolves by  $\times 20$  from  $z \sim 4$  until today; this is consistent with  $\hat{M}_{\text{IMF}} \propto (1+z)^2$  as proposed here. Note that his analysis only applies to stars formed in early-type cluster galaxies, so is not necessarily applicable to all galaxies. Making the assumption that it is widely applicable, Van Dokkum (2007) determined a cosmic stellar mass growth rate that is broadly consistent with the evolving IMF case shown in Figure 5 (though in detail it is more peaked towards lower  $z$ ; see his Fig. 13). While many uncertainties are present in his analysis, it is reassuring that similar IMF evolution is inferred from a completely independent line of argument.

The ultimate test of an evolving IMF is to directly measure the stellar mass buildup in the Universe. Recently, Perez-Gonzalez et al. (2007) and Elsner et al. (2007) did just that using deep *Spitzer*/IRAC observations that probe near-IR light all the way out to  $z \sim 3+$ . Their data of cosmic stellar mass growth rates as a function of redshift are shown as the magenta and red data points in Figure 5, respectively (assuming a Salpeter IMF). These data points were obtained by differentiating the stellar mass densities as a function of redshift published in those papers. Although those two samples are independent, both show that the stellar mass growth rate is significantly different than that inferred from the observed cosmic star formation history, particularly at  $z \gtrsim 2$ . Perez-Gonzalez et al. (2007) notes that this may favor an IMF weighted more towards massive stars at high redshifts. They mention that utilizing a Chabrier or Kroupa IMF will lessen the discrepancy, but such a reduction is essentially by a fixed factor at all redshifts (modulo minor stellar evolution differences), so even if the normalization is adjusted to agree with the cosmic SFH at  $z = 0$ , it would still yield significantly lower SFRs at  $z \gtrsim 2$ .

A major uncertainty in this comparison is that Perez-Gonzalez et al. (2007), in order to integrate up the total stellar mass, assumed a Schechter (1976) faint-end slope that is fixed at  $\alpha = -1.2$ , even though they cannot directly constrain  $\alpha$  at  $z \gtrsim 2$ . They justify this assumption by noting that there is no evidence for a change in  $\alpha$  from  $z \sim 0 \rightarrow 2$ . But if  $\alpha$  becomes progressively steeper with redshift, as suggested by Fontana et al. (2006), it could also reconcile these observations. Note that it would have to steepen quickly, e.g. to  $\alpha \approx -1.8$  at  $z \sim 3$  to reconcile the  $\sim \times 3$  difference, significantly faster than  $d\alpha/dz = -0.082$  observed by Fontana et al. (2006) out to  $z \sim 4$ . Elsner et al. (2007) also found that  $\alpha$  does increase with redshifts slightly in one form of their analysis, though again not by enough to explain the discrepancy. Still, the difficulty of constraining the faint-end slope at high- $z$ , along with inherent uncertainties in SED fitting, mean that these observations cannot be considered definitive proof of a varying IMF. But at least at face value the agreement with the evolving IMF case is quite good.

Wilkins, Trentham & Hopkins (2007) independently determined the cosmic stellar mass growth rate by compiling observations of stellar mass densities from the literature for  $z \sim 0-4$  (note that they do not include the Perez-Gonzalez et al. or Elsner et al. data). Carefully accounting for various different systematics, they

fit a Cole et al. (2001) form to the implied cosmic star formation history (including effects of stellar mass loss), and determine  $\dot{\rho}_* = (0.014 + 0.11z)h[1 + (z/1.4)]^{2.2}$ , in  $M_\odot \text{yr}^{-1} \text{Mpc}^{-3}$ . This relation is shown as the cyan dashed line in Figure 5; it has been adjusted by +0.22 dex to change from the IMF they assumed (similar to Chabrier) to a Salpeter IMF. It agrees well with other observations, and lies significantly below the star formation rate inferred from high-mass SFR indicators at high- $z$ . Wilkins et al. suggest that an evolving IMF may be the cause, and propose their own form of IMF evolution to explain it. As seen in Figure 5, the agreement with the evolving IMF proposed here is quite good.

In short, an evolving Kroupa IMF does not obviously introduce any large inconsistencies with high- $z$  galaxy data, and moreover may be preferred based on low-redshift fossil light considerations and observed cosmic stellar mass growth rates. Hence the amplitude evolution of the  $M_*$ –SFR relation may be yet another example of an observation of stellar mass buildup that suggests that the IMF is weighted towards more massive stars at higher redshifts.

## 8 SUMMARY

Implications of the observed stellar mass–star formation rate correlation are investigated in the context of current theories for stellar mass assembly. The key point, found here and pointed out in previous studies (Daddi et al. 2007; Elbaz et al. 2007), is that the amplitude of the  $M_*$ –SFR relation evolves much more rapidly since  $z \sim 2$  in observations than in current galaxy formation models. It is shown here that this is true of both hydrodynamic simulations and semi-analytic models, and is broadly independent of feedback parameters. In contrast, the slope and scatter of the observed  $M_*$ –SFR relation are in good agreement with models.

The tight  $M_*$ –SFR relation with a slope near unity predicted in models is a generic result owing to the dominance of cold mode accretion, particularly in early galaxies, which produces rapid, smooth and relatively steady infall. The slow amplitude evolution arises because star formation starts at  $z \gtrsim 6$  for moderately massive star-forming galaxies and continues at a fairly constant or mildly declining level to low- $z$ . The large discrepancy in amplitude evolution when compared to observations from  $z \sim 2 \rightarrow 0$  hints at some underlying problem either with the models or with the interpretation of data. A convenient parameterization of the problem is through the star formation activity parameter,  $\alpha_{\text{sf}} \equiv (M_*/\text{SFR})/(t_H - 1 \text{ Gyr})$ , where  $t_H$  is the Hubble time. A low value corresponds to a starbursting system, a high value to a passive system, and a value near unity to system forming stars constantly for nearly a Hubble time. In models,  $\alpha_{\text{sf}}$  remains constant around unity from  $z = 0 - 4$ , whereas in observations it rises steadily from  $\lesssim 0.2$  at  $z \sim 2$  to close to unity at  $z \sim 0$ .

Several ad hoc modifications to the theoretical picture of stellar mass assembly are considered in order to match  $\alpha_{\text{sf}}$  evolution, but each one is found to be in conflict with other observations of high-redshift galaxies. Bursts seem unlikely given the low scatter in  $M_*$ –SFR. Delaying galaxy formation to match  $\alpha_{\text{sf}}$  results in such a low redshift for the onset of star formation, it is in conflict with observations of star forming galaxies at earlier epochs. Hiding a large population of galaxies as passive runs into difficulty when compared to direct observations of the passive galaxy fraction in mass-selected samples at  $z \sim 2$ . Having an exponentially growing phase of star formation or “staged” galaxy formation are perhaps the most plausible solutions from an observational viewpoint, but it is difficult to understand theoretically how such sce-

narios can arise within hierarchical structure formation. Hence if observed star-forming galaxies are quiescently forming stars and represent the majority of galaxies at  $z \sim 2$ , as other observations seem to suggest, then it is not easy to accommodate the low  $\alpha_{\text{sf}}$  inferred from the  $M_*$ –SFR amplitude.

Systematic uncertainties in  $M_*$  or SFR determinations could bias results progressively more at high- $z$  in order to mimic evolution in  $\alpha_{\text{sf}}$ . Various currently debated sources are considered, such as the contribution to near-IR light from TP-AGB stars, extinction corrections, assumptions about star formation history, AGN contamination, and PAH emission calibration. It may be possible to concoct scenarios whereby several of these effects combine to mimic  $\alpha_{\text{sf}}$  evolution, but there are no suggestions from local observations that such scenarios are to be expected. Hence if systematic effects are to explain the low  $\alpha_{\text{sf}}$  at high- $z$ , it would imply significant and unexpected changes in tracers of star formation and stellar mass between now and high redshifts.

A solution is proposed that the stellar initial mass function becomes increasingly bottom-light to higher redshifts. Several lines of arguments are presented that vaguely or circumstantially favor such evolution, though no smoking gun signatures are currently known. A simple model of IMF evolution is constructed, based on the ansatz by Larson (1998) that the minimum temperature of molecular clouds is reflected in the characteristic mass of star formation ( $\hat{M}_{\text{IMF}}$ ) where the mass contribution per logarithmic mass bin is maximized. The minimum temperature may increase with redshift owing to a hotter cosmic microwave background, more vigorous star formation activity, or lower metallicities within early galactic ISMs. In order to reconcile the observed  $\alpha_{\text{sf}}$  evolution with theoretical expectations of no evolution, an evolving IMF of the form

$$\begin{aligned} \frac{dN}{d \log M} &\propto M^{-0.3} \text{ for } M < \hat{M}_{\text{IMF}}, \\ &\propto M^{-1.3} \text{ for } M > \hat{M}_{\text{IMF}}; \\ \hat{M}_{\text{IMF}} &= 0.5(1+z)^2 M_\odot \end{aligned}$$

is proposed. The exponent of  $\hat{M}_{\text{IMF}}$  evolution is constrained by requiring no  $\alpha_{\text{sf}}$  evolution, through careful modeling with the PE-GASE.2 population synthesis code. While the exact form of IMF evolution is not well constrained by present observations, what is required is that the IMF has progressively more high-mass stars compared to low-mass at earlier epochs, and that this IMF applies to the majority of star forming galaxies at any epoch. It is worth noting that this evolving IMF is only constrained out to  $z \sim 2$  from the  $M_*$ –SFR relation, though it yields predictions that are consistent with other observations out to  $z \sim 4$ . Extrapolating such evolution to higher redshifts is dangerous, since no observational constraints exist and the precise cause of IMF evolution is not understood.

Implications of such an evolving IMF are investigated. By leaving the high-mass end of the IMF unchanged, recent successes in understanding the connections between high-mass star formation, feedback, and metal enrichment are broadly preserved. The cosmic stellar mass accumulation rate would be altered compared to what is inferred from cosmic star formation history measurements using a standard IMF. It is shown that an evolving IMF is at face value in better agreement with direct measures of cosmic stellar mass assembly (Perez-Gonzalez et al. 2007; Elsner et al. 2007; Wilkins, Trentham & Hopkins 2007). Furthermore, the evolving IMF goes towards relieving the generic tension between present-day fossil light measures versus observations of the cosmic star formation history. In particular, the paunchy IMF favored by

Fardal et al. (2007) in order to reconcile the observed cosmic star formation history, present-day  $K$ -band luminosity density, and extragalactic background light constraints, is qualitatively similar to the cumulative IMF of all stars formed by today in the evolving IMF case. Individually, each argument has sufficient uncertainties to cast doubt on whether a radical solution such as an evolving IMF is necessary. But taken together, the  $M_*$ -SFR relation adds to a growing body of circumstantial evidence that the ratio of high-mass to low-mass stars formed is higher at earlier epochs.

It is by no means clear that IMF variations are the only viable solution to the  $M_*$ -SFR dilemma. The claim here is only that an evolving IMF is an equally (un)likely solution as invoking unknown systematic effects or carefully crafted star formation histories in order to explain  $M_*$ -SFR evolution. It is hoped that this work will spur further efforts, both observational and theoretical, to investigate this important issue.

Future plans include performing a more careful analysis of the evolving IMF in terms of observable quantities, in order to accurately quantify the impact of such IMF evolution on the interpretation of UV, near-IR, and mid-IR light. Also, it is feasible to incorporate such an evolving IMF directly into simulation runs to properly account for gas recycling in such a scenario. Finally, including some feedback mechanism to truncate star formation in massive systems may impact the  $M_*$ -SFR relation in some way, so this will be incorporated into the hydro simulations.

Observationally, pushing SFR and  $M_*$  determinations to higher redshifts is key; a continued drop in  $\alpha_{\text{SF}}$  to  $z \sim 3$  would rapidly solidify the discrepancies with current models, and would also generate stronger conflicts with other observations of high- $z$  galaxies. Assessing the AGN contribution and extinction uncertainties from high- $z$  systems is critical for accurately quantifying the light from high-mass star formation, for instance through the use of more direct star formation indicators such as Paschen- $\alpha$ . Pushing observations further into the mid-IR such as to  $70\mu$ , past the PAH bands at  $z \sim 2$ , would mitigate PAH calibration uncertainties in SFR estimates. Obtaining a large sample of spectra for typical high- $z$  star-forming systems (like cB58; Pettini et al. 2000) would more accurately constrain the SED than broad-band data. All of these programs push current technological capabilities to their limits and perhaps beyond, but are being planned as facilities continue their rapid improvement.

In summary, owing to the robust form of star formation histories in current galaxy formation models, the  $M_*$ -SFR relation represents a key test of our understanding of stellar mass assembly. Current models reproduce the observed slope and scatter remarkably well, but broadly fail this test in terms of amplitude evolution. Whether this reflects some fundamental lack of physical insight, or else some missing ingredient such as an evolving IMF, is an issue whose resolution will have a significant impact on our understanding of galaxy formation.

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